2015 Che2410 – Homework Assignment #4
Due on Nov. 13th at midnight

1. Solve the following differential equations, please show your work.

a) $y'' + 6y' + 5y = 0$

b) $y'' + 6y' + 9y = 0$

c) $y'' + 6y' + 13y = 0$

d) $y'' - 3y' + 2y = xe^{x} + xe^{-x}$

e) $y'' + 4y' + 4y = e^{-2x}/x^2$

f) $(x-1)y'' - xy' + y = 0$

2. Solve (if possible) the following problems. If necessary, indicate a solvability condition. Comment on the uniqueness of the solution.

(Note that, if you can show using the Fredhold Alternative Theorem that there is no solution, you do not need to do any more work on the solution. Similarly, you may want to use a theorem proves that a solution to the initial-value problem is unique).

a) $y''' - 2y'' + y' = 1 + xe^{x} \quad (x > 0); \quad y(0) = 0, y'(0) = 0, y''(0) = 1$

b) $e^{x}y''' + \sin x\, y'' + \cos x\, y' + x^6 y = 0 \quad (x > 0); \quad y(0) = 0, y'(0) = 0, y''(0) = 0$

c) $xy'' - (x+1)y' + y = x^2 e^{2x} \quad (x > 1); \quad y(1) = 0, y'(1) = e$

Hint: $y(x) = x + 1$ is a solution to the homogeneous equation

You may run into difficult integrals like $\int e^x x/(x+1)^2 dx$. For this integral one has to use a trick together with integration by parts:

$$\begin{aligned}
\int e^x \frac{x}{(x+1)^2} dx &= \int e^x \frac{x+1}{(x+1)^2} dx - \int e^x \frac{1}{(x+1)^2} dx \\
&= \int e^x \frac{1}{x+1} dx - \left(e^x \frac{-1}{x+1} + \int e^x \frac{1}{x+1} dx \right) \\
&= e^x \frac{1}{x+1}.
\end{aligned}$$

i.e. while the integration by parts leads only to another difficult integral, it turns out that in this case the difficult integral cancels the other difficult integral.