2015 Che2410 – Homework Assignment #5
Due on Dec. 4th at midnight

1. Find solutions that are valid for the following differential equation near $x = 0$,

$$x^2y'' - \frac{1}{2}xy' + \frac{1}{2}(\alpha + \beta x)y = 0$$

a) Find solutions when $\beta = 0$ and $\alpha \leq 9/8$.

b) Use Frobenius theory to obtain the general solution for $\alpha = 1$. Write the power series for $y^+ and\ y^-$ in terms of elementary functions.

c) Once you have the general solution from part (b), consider the limit of $\beta \ll 1$ perform a Taylor expansion in β (i.e., find $y^+ = y_0^+ + \beta y^+$ and similarly for y^-). How do the solutions in this limiting case compare to the solutions in part (a)?

2. Consider the equation:

$$y'' - xy = 0$$

This equation is called Airy's equation. It is important for modeling quantum-mechanical particles hitting walls defined by smooth potentials. Use Frobenius theory to obtain the general solution this equation. The two homogeneous solutions are called Airy functions and cannot be expressed in terms of other elementary functions.

3. Determine the two values of the constant α for which all solutions of

$$xy'' + (x-1)y' - \alpha y = 0$$

can be written as a power series (i.e., $y = x^s \sum_{n=0}^{\infty} A_n x^n$).