

2016 Che2410 – Homework Assignment #1

Due on Sept. 27th at 6:30pm

1. (10 pts) Classify the following differential equations. If applicable, state whether the equation is linear/nonlinear, ordinary/partial, homogeneous/inhomogeneous, the order, and whether the equation is elliptic, hyperbolic, or parabolic, and under what conditions (if any).

a) $f''' = f$

b) $y + \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} = x$

c) $f_t f_{xx} = 1$

d) $\sqrt{y(x)} + y'(x) + xy''(x) = 5x$

e) $(y + 1)dy = (4 - 4xy)dx$

f) $f_{xx} + f_{xy} = f_{yy}$

g) $\frac{\partial^2 y}{\partial x^2} + x \frac{\partial^2 y}{\partial t^2} + t \frac{\partial^2 y}{\partial x \partial t} = y$

h) $\nabla^2 \phi = f$

i) $\sin(f) = f'$

j) $\tan(xy) \frac{dz}{dx} + \frac{dz}{dy} = 0$

2. (10 pts) Solve the following 1st order differential equations:

a) $x^3 \frac{df}{dx} + 4f = 2f - 5$

b) $8xf + (x^2 + 4)f' = x$

3. (10 pts) Derive a formula is 4th order in error for:

(a) a forward difference operator

(b) a central difference operator

In each case, determine the leading order term in the truncation error.

4. (10 pts) Derive a 3rd order approximation to the second derivative using a backward difference operator. Determine the leading order term in the truncation error.

5. (10 pts) Consider the equation:

$$u_t = u_{xx} + u_{xxxx}$$

Is this a well-posed problem? Can it be solved using numerical methods?

(Some background: https://en.wikipedia.org/wiki/Well-posed_problem)