2016 Che2410 – Homework Assignment #2
Due on Oct. 18th at 6:30pm

1. (10 pts) Solve the following differential equation numerically:

$$f_t = \cos(t)\,, \qquad f(0) = 2, \qquad 0 \le t \le 5$$

a) Find the exact solution.

b) Choose a scheme so that your solution at $t = 5$ is within 1% of the exact answer. Plot your numerical solution vs. the exact solution and report the value of both at $t = 5$.

c) Repeat part (b) but change the timestep to reduce the error by a factor of 10.

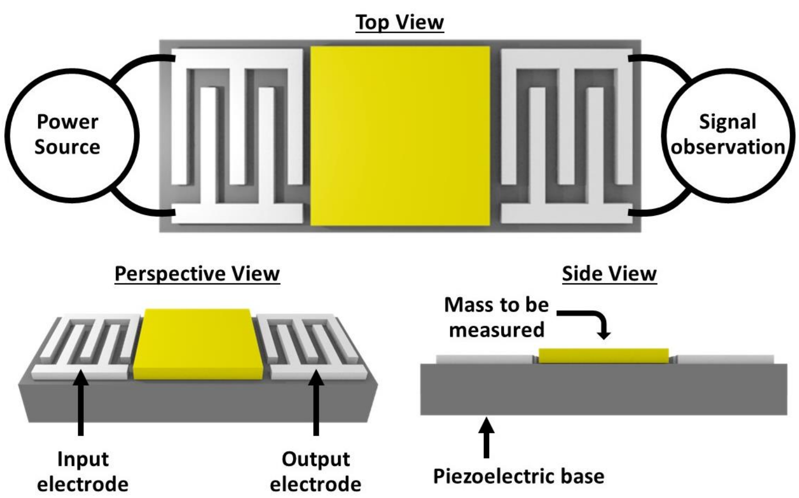
2. (20 pts) Solve the following differential equation numerically. Note that this is a *nonlinear* differential equation – none of the techniques for solving differential equations analytically in this class will work on this equation.

$$f_t = \cos(t)\, f^2, \qquad f(0) = 1, \qquad 0 \le t \le 1$$

a) Choose a scheme to solve this equation numerically. Since it may not be possible to compare your answer against an exact answer, keep making your timestep smaller by factors of 2 until your solution at $t = 1$ doesn't change by more than 1%. Plot your best numerical solution and report the value at $t = 1$.

b) Is this differential equation well posed? Can you converge to a solution at $t = 1.5$? Show your work.

3. (30 pts) Consider a surface acoustic wave sensor.



In these sensors, a wave is generated at the input electrode, which travels through a sensor material (yellow) and arrives at the output electrode. Depending on the mass of the sensor material, the wave will travel more quickly or more slowly. This traveling wave phenomenon can be modeled by the following partial differential equations:

$f_t = -0.25\, f_x, \qquad |x| < 4, \qquad -10 < x < 10,$ Sensing material region

$f_t = -f_x, \qquad |x| \geq 4, \qquad -10 < x < 10,$ Input/output electrode region

(continued from #3 above)

Solve these partial differential equations numerically using the Lax-Wendroff scheme. Assume the wave has the form of a Gaussian with $\sigma = 5$ and that at $t = 0$ the wave is centered at $x = -9$. Apply periodic boundary conditions (even though it is unrealistic) to handle the edges.

a) Why are periodic boundary conditions unrealistic for this problem? At what value of t does the wave return to its starting point?

b) After finding an appropriate Δt and Δx (justify your choice), plot the solution at various points in time from when it is on the input electrode to when it arrives at the output electrode.

c) Describe what happens to the wave in the different regions: How is the peak changing? How is the width changing? How is the wave speed changing?

d) What aspects of your solution are probably "real" vs. what parts are probably due to approximation error?

e) (5 pts bonus) Change the periodic boundary conditions to reflecting boundary conditions. Is this more physical realistic? Why, why not? How long does it take for the wave to return to its starting point?